ANANDALAYA

PERIODIC TEST – 2

Class : XII

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Subject: Mathematics M.M: 80

Date : 27 – 09 –2022 Time: 3 Hours

General Instructions:

1. The question paper contains two parts A and B.
2. Both parts A and B have internal choices.

Part – A

1. It consists two sections –I and II.
2. Section I has 16 questions of 1 mark each. Internal choice has been provided in 5 questions.
3. Section II has 2 questions on case study. Each case study has 5 case based sub parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B

1. It consists three sections –III, IV and V.
2. Section – III comprises of 10 questions of 2 marks each,
3. Section – IV comprises of 7 questions of 3 marks each,
4. Section – V comprises of 3 questions of 5 marks each.
5. Internal choice has been provided in 3 questions of 2 marks, 2 questions of 3 marks and 3 questions of 5 marks. You have to attempt only one of the alternatives in all such questions.

PART - A

Section –I

Questions in this section carry 1 mark each.

1. Evaluate: $\int x\, sinx\, dx$. **OR** Evaluate: $\int sin^3x\, cosx\, dx$. (1)

2. Check if the function $f: N \to N$ defined by $f(x) = 2 - 3x$ is one-one or not? (1)

3. If the relation R in the set {1, 2, 3} given by R ={(1,2),(2,1),(1,1)} . Then state if R is transitive relation, justify your answer. (1)

4. Simplify : (1)
$$sec\theta \begin{bmatrix} tan\theta & sec\theta \\ sec\theta & -tan\theta \end{bmatrix} - tan\theta \begin{bmatrix} sec\theta & tan\theta \\ tan\theta & -sec\theta \end{bmatrix}$$

5. If a matrix has 18 elements, then how many possible orders it can have? (1)

6. Let a relation R in a set A contains $(a_{1,}a_2) \in R$. If R is a symmetric relation, then write the element which must be in R. (1)

7. Find the value of: $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{x^2} \tan\frac{1}{x}\, dx$ (1)

8. If $y = e^{\mathrm{a}\cos^{-1} 3x}$, then find $\frac{dy}{dx}$. (1)

9. If $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$, then the value of y is______. (1)

10. Evaluate: $\int_{-2}^{2}(x^3 - 1)dx$. (1)

11. Evaluate: $\cos^{-1}(1) + \sin^{-1}\left(\frac{-1}{2}\right)$. (1)

OR

What is the principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$.

12. If $R = \{(x, y): x + 2y = 8\}$ is a relation on N, write the range of R. (1)

13. If A is a square matrix of order 2 such that, $adj.(3A) = \mu(adj.A)$ then find the value of μ. (1)

OR

If $A = \begin{bmatrix} 8 & 2 \\ 3 & 2 \end{bmatrix}$, then find the value of $|adj.A|$.

14. Find $\int \frac{x-1}{(x-2)(x-3)} dx$. (1)

OR

Find $\int e^x(cosx - sinx)cosec^2 x dx$.

15. If $\sin^{-1} x + \cos^{-1} \frac{1}{2} = \frac{\pi}{2}$, then find x. (1)

16. Write the derivative of $sinx$ with respect to $cosx$. (1)

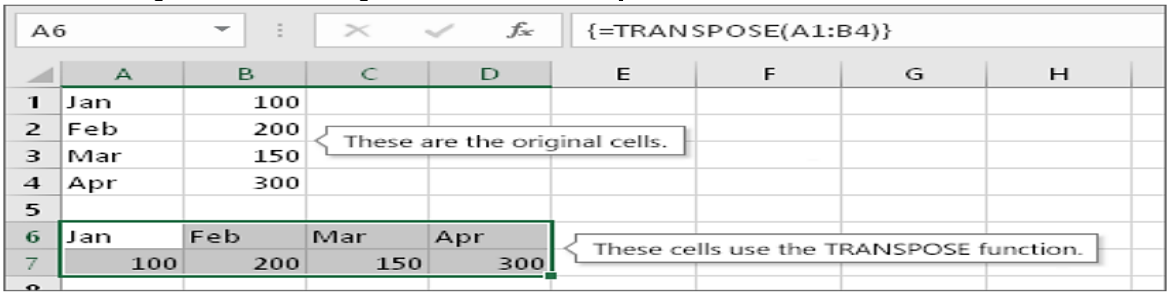
OR

If $f(x) = sin3x - cos3x$, find $f'\left(\frac{\pi}{6}\right)$.

Section II

Questions in this section carry 1 mark each.
Both the Case study based questions are compulsory. Attempt any 4 sub-parts from each question 17 (i – v) and 18 (i – v).

17. Samay is working with Excel. He needs to switch or rotate cells. He can do this by selecting the options of copying, pasting or transpose option. But by doing this the data will be duplicated. To avoid this he is using a formula {=TRANSPOSE (A1:B4)} which takes the cells A1 through B4 and arranges them horizontally as shown below. (4)



i) A square matrix A is expressed as sum of symmetric and skew symmetric matrices, and then symmetric part of A is

(A) $\frac{1}{2}(A + A^T)$ (B) $\frac{1}{2}(A - A^T)$ (C) $\frac{1}{2}(A^T - A)$ (D) None of them

ii) A square matrix A is expressed as sum of symmetric and skew symmetric matrices, and then skew-symmetric part of A is

(A) $\frac{1}{2}(A + A^T)$ (B) $\frac{1}{2}(A - A^T)$ (C) $\frac{1}{2}(A^T - A)$ (D) None of them

iii) Symmetric part of $A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 2 & 1 \\ 4 & 5 & 7 \end{bmatrix}$

(A) $\begin{bmatrix} 1 & \frac{5}{2} & \frac{9}{2} \\ \frac{5}{2} & 2 & 3 \\ \frac{9}{2} & 3 & 7 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -\frac{5}{2} & -\frac{9}{2} \\ -\frac{5}{2} & 2 & 3 \\ -\frac{9}{2} & -3 & 7 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & \frac{5}{2} & \frac{9}{2} \\ \frac{5}{2} & 0 & 3 \\ \frac{9}{2} & 3 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & -\frac{5}{2} & -\frac{9}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{9}{2} & -3 & 0 \end{bmatrix}$

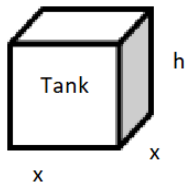
iv) Skew- symmetric part of $A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 2 & 1 \\ 4 & 5 & 7 \end{bmatrix}$

(A) $\begin{bmatrix} 0 & \frac{-1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & -2 \\ -\frac{1}{2} & 2 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 0 & -2 \\ \frac{1}{2} & 2 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{bmatrix}$

v) When writing square matrix A as sum of symmetric and skew-symmetric matrices, is symmetric and skew symmetric matrices are unique?

(A) Yes (B) No

18. A Social organization working for welfare of the society, decides to conserve water in a village. For this purpose, the organization proposes to construct an open water tank having a square base, by using a metal sheet. The area of the metal sheet is to be least and water to be stored in the tank is 4000 cubic metres. (4)



Using the information given above, answer the following :

i) Which of the following represents area (A) of the metal sheet to be used for making the open water tank?

(A) $4hx$ (B) $2hx$ (C) $2x + h$ (D) $x^2 + 4hx$

ii) Volume of the water stored in the tank is given by

(A) $\frac{x^2}{h} + X = 4000$ (B) $xh^2 = 4000$ (C) $hx^2 = 400$ (D) $x^2h = 4000$

iii) $\frac{dA}{dx}$ equals

(A) $2x + \frac{4}{h}$ (B) $x + 4h + 4x\frac{d(h)}{dx}$ (C) $2x - \frac{16000}{x^2}$ (D) None of them

iv) Least area of the tank occurs when x equals

(A) $20m$ (B) $20cm$ (C) $20units$ (D) $40m$

v) Least value of A is

(A) $1300m^2$ (B) $1200cm^2$ (C) $1200m^2$ (D) $1100m^2$

PART – B

Section – III

Questions in this section carry 2 marks each.

19. If $y = ae^{2x} + be^{-x}$, then show that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ (2)

20. $\int \frac{\cos(x+a)}{\sin(x+b)}dx$ **OR** $\int_{\frac{-\pi}{4}}^{0} \frac{1+tanx}{1-tanx}dx$ (2)

21. Find the value of p for which the function $f(x) = \begin{cases} \frac{1-cos4x}{x^2}, & x \neq 0 \\ p, & x = 0 \end{cases}$ is continuous at $x = 0$. (2)

22. If $f: R \rightarrow R$ be the function defined by $f(x) = 4x^3 + 7$, show that the function is a bijection. (2)

23. Find: $\int \sqrt{2x - x^2}dx$ **OR** Find: $\int \frac{dx}{x^2+4x+8}$. (2)

24. A balloon , which always remains spherical, has a variable diameter $\frac{2}{3}(3x + 1)$. Find the rate of change of its volume with respect to x. (2)

25. Write the value of $\int \frac{x+cos6x}{3x^2+sin6x}dx$. (2)

26. Find the maximum and minimum value of $f(x) = x^3 - 3x^2 + 6x - 100$. (2)

27. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, find the value of x. (2)

OR

If $x \in N\ and \begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, find the value of x.

28. Find $\int \frac{sin^6x}{cos^8x}dx$. (2)

Section-IV

Questions in this section carry 3 marks each.

29. Find the intervals in which the function is $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$. (3)
(a) strictly increasing
(b) strictly decreasing.

30. Show that of all the rectangles of given area, the square has the smallest perimeter. (3)

OR

Prove that the area of a right angled triangle of given hypotenuse is maximum when the triangle is isosceles.

31. Find the matrix X if $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\\ 11 & 10 & 9\end{bmatrix}$ (3)

32. Prove that the function f given by $f(x) = |x-5|, x \in R$ is continuous but not differentiable at $x = 5$. (3)

33. Find $\frac{dy}{dx}, if\ y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$. (3)

OR

$If\ x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{logx}{[log(xe)]^2}$.

34. Let N be the set of natural numbers and R be the relation on $N \times N$defined by $(a,b)R(c,d)\ if\ ad(b+c) = bc(a+d)$. Show that R is an equivalence relation. (3)

35. For the following matrices $A = \begin{bmatrix}1\\ -4\\ 3\end{bmatrix}$and $B = [-1 \quad 2 \quad 1]$, verify that $(AB)' = B'A'$. (3)

Section-V

Questions in this section carry 5 marks each.

36. Differentiate the following function with respect to x. (5)
$x^{sinx} + (sinx)^{cosx}$

OR

If $y = (\sec^{-1} x)^2$, then show that $x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} = 2$.

37. Evaluate: $\int \frac{x^2+1}{(x^2+4)(x^2+25)}dx$ **OR** Evaluate: $\int_0^\pi \frac{x\ tanx}{secx+tanx}dx$ (5)

38. If $A = \begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = O$. Hence find A^{-1}. (5)

OR

Using matrix method, solve the following system of equations:
$2x - 3y + 5z = 13$,
$3x + 2y - 4z = -2$,
$x + y - 2z = -2$.
Hence find the value of x^{yz}.